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**A Model for Cold Region Hydrology:**  
variable-density, two-phase  
groundwater flow coupled with heat  
transfer and phase change  
My Code Has No Name (yet)!

# Physical Equations

Energy Equation  
(temperature field)

$$\rho C_p \frac{\partial T}{\partial t} + \rho C_p u \cdot \nabla T = K \nabla^2 T + \varphi_i$$

$i = \text{ice}$

Latent Heat Treatment  
(heat source/sink)

$$\varphi_i = \rho_i L_i \frac{\partial S_i}{\partial t} = \rho_i L_i \frac{\partial S_i}{\partial T} \frac{\partial T}{\partial t} \quad \text{e.g., } S_i = \phi_i$$

Darcy's Law with Buoyancy  
(velocity field)

$$u_j = -\frac{k_j^r k(\phi)}{\mu_j} \nabla [P - \rho_j g z] \quad j = \text{phase subscript}$$

Fluid/Gas Cons. of Mass  
(fluid/gas saturation)

$$\frac{\partial S_j}{\partial t} + \nabla \cdot u_j = \Phi_j \quad \sum_j (\nabla \cdot u_j - \Phi_j) = 0$$

Solute Field  
(salinity, etc)

$$\frac{\partial C}{\partial t} + \nabla \cdot (uC) = D \nabla^2 C + Q_c$$

# Assumptions

- Darcy flow  $\rightarrow Re \ll 1$
- Water phase change is  $f(\text{Temp}, \text{Press}, \text{Sal})$  according to UNESCO, 1983
- Volume change between ice/water not accounted for
- Density & specific heat in grid cell calculated with volume average
- Thermal conductivity in grid cell calculated with mixture model
- Relative permeability defined by Corey curves
- Absolute permeability is a function of porosity (which accounts for ice content, e.g. ice permafrost)
- Saturated conditions (not equipped for unsaturated zone)

# Method of Solution

- Finite Volume Method (FVM) used to discretize the equations

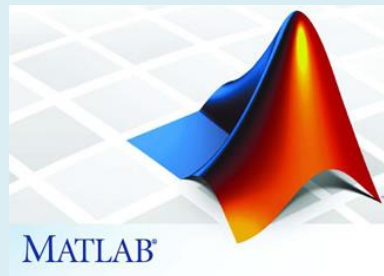
e.g.,

$$\int_V \int_t \sum_j (\nabla \cdot u_j - \Phi_j) = 0$$

- C++ used to code up the model
- Equations are solved using pre-conditioned iterative methods using the PETSc library



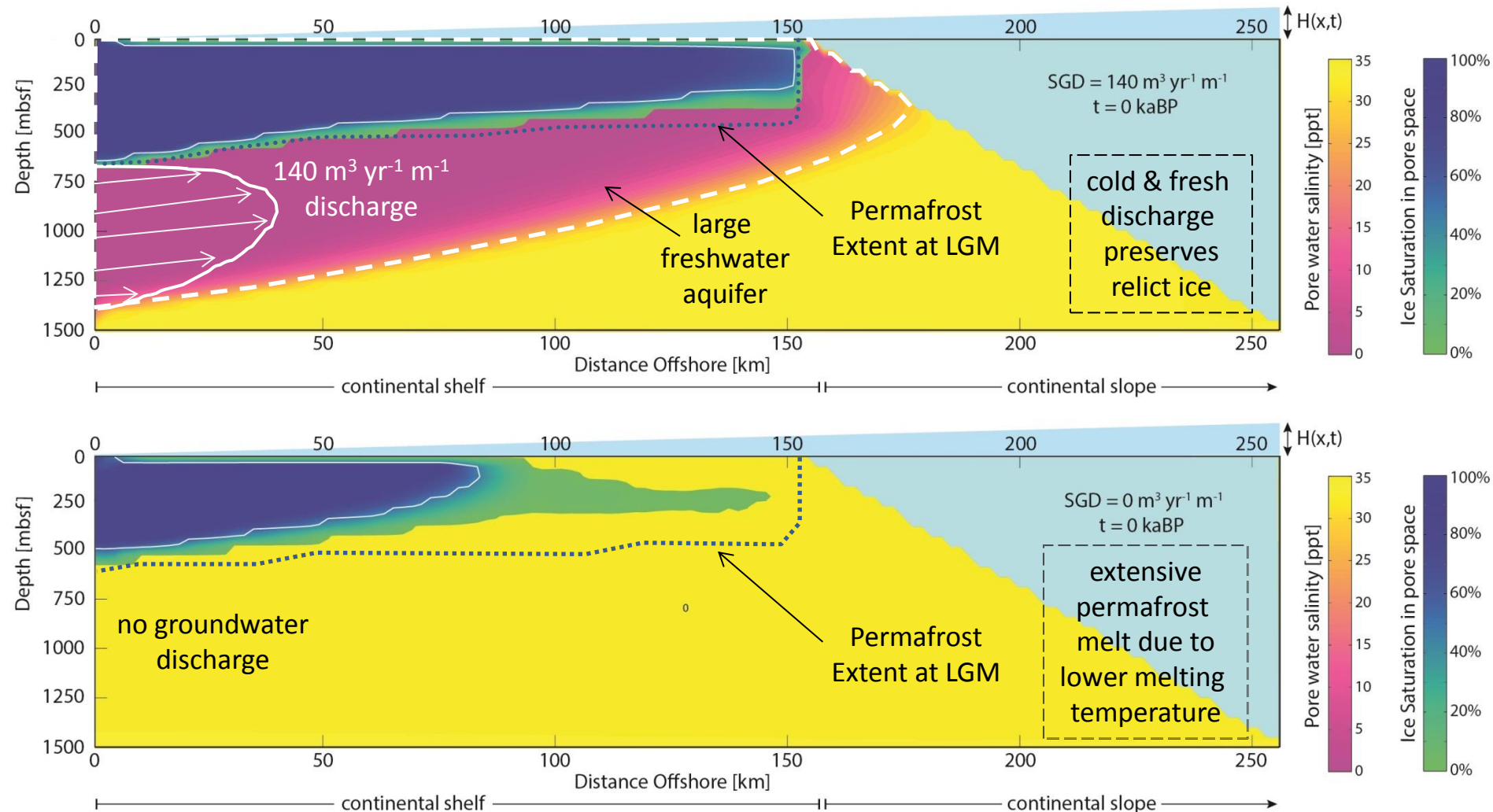
- Post-processing and visualization done with MATLAB and Visit software



# Model Applications

Investigating the effect of submarine groundwater discharge on relict Arctic submarine permafrost:

Frederick & Buffett (2014), *JGR, in review*



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# Model Applications

Frederick & Buffett (2014), *JGR Earth Surface*

Investigating methane gas venting through several taliks in relict Arctic submarine permafrost:

\*(the taliks are formed by heat input from river at surface, red line)

methane gas (yellow)

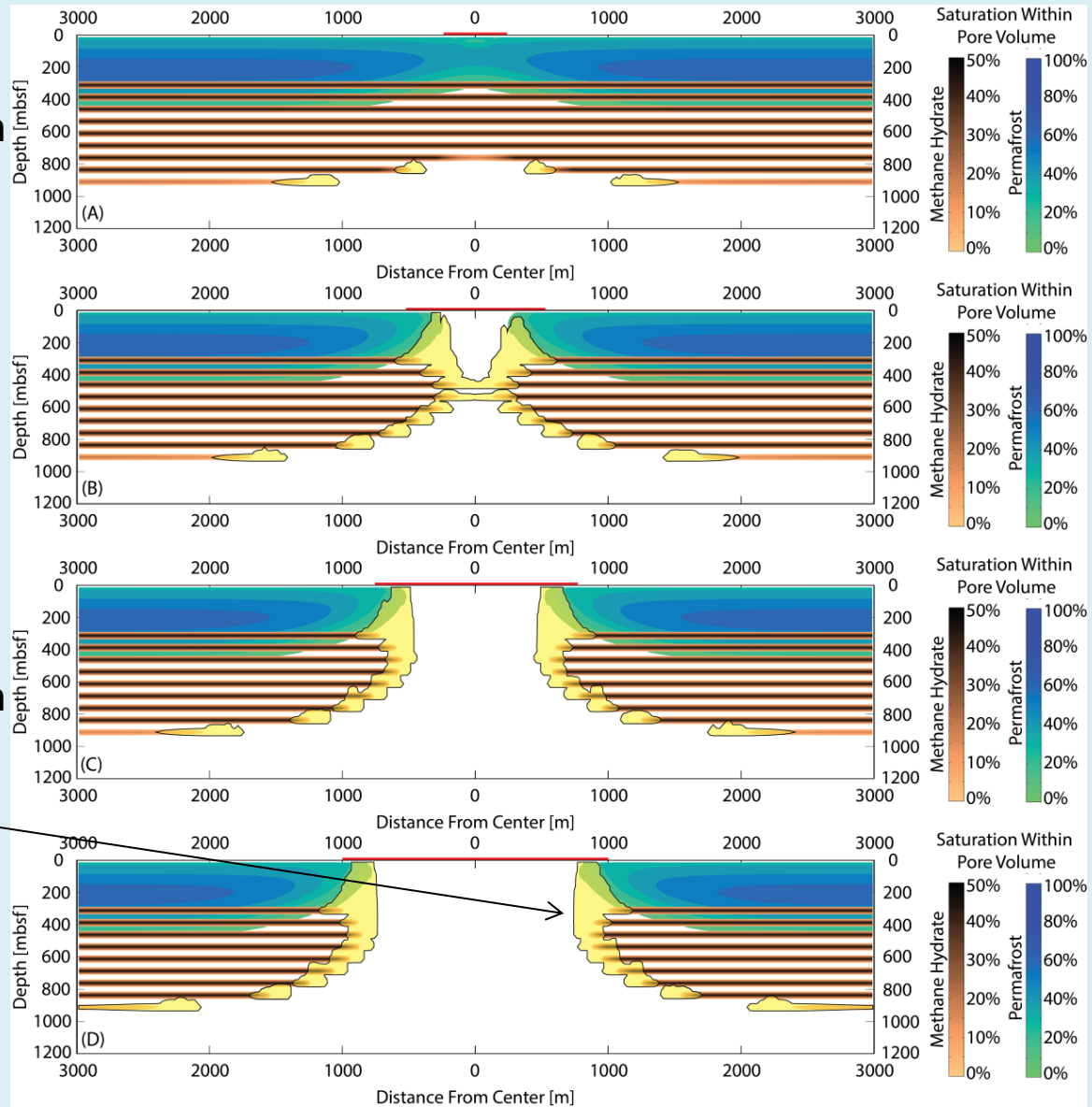
river\* width:

0.5 km

1.0 km

1.5 km

2.0 km



# Benchmarks

- I can run all benchmarks except the unsaturated ones
- Timeline to complete benchmark cases is flexible
- I am interested in a joint publication

Please feel free to contact me anytime!

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