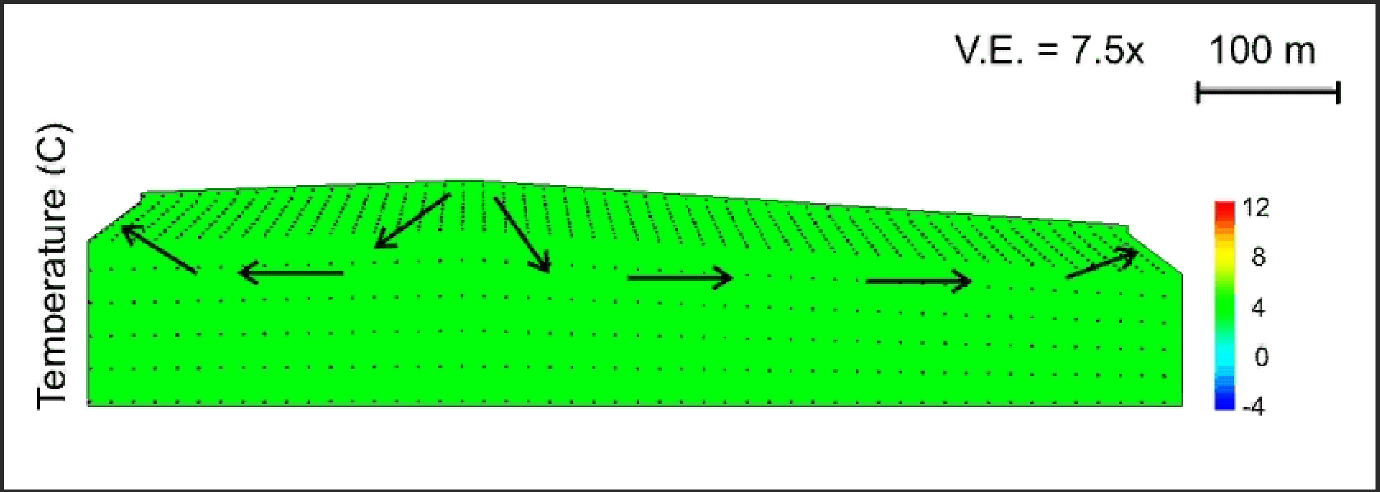
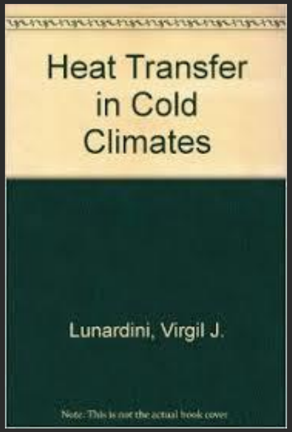
Cold Regions Coupled Thermohydraulic Code Benchmarking (T1 and TH1): SUTRA Results

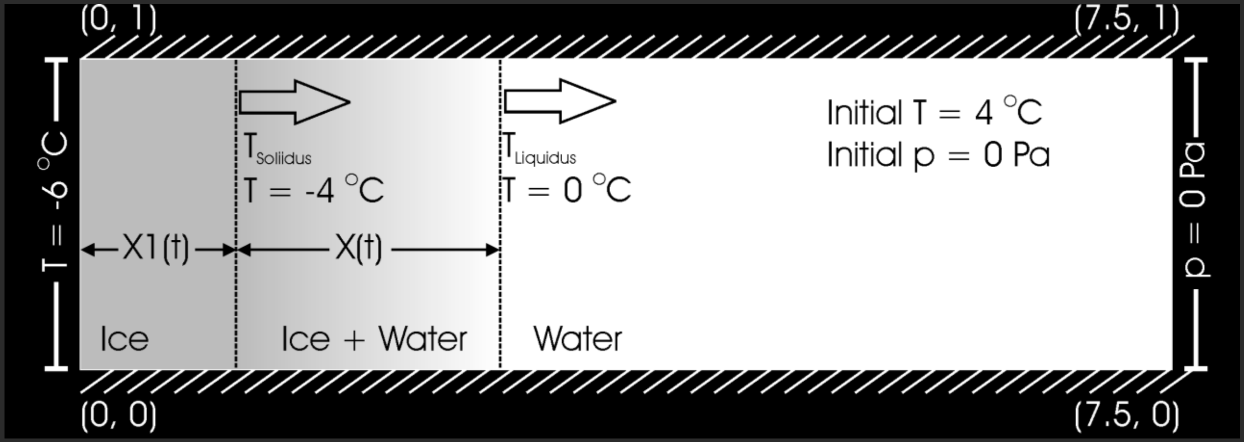


Jeffrey McKenzie, Barret Kurylyk
Clifford Voss, and Kerry MacQuarrie

Introduction: Benchmark 1 (T1)



1. **Proposed by Lunardini (1981)**
   (*Heat Transfer in Cold Climates, p. 393-396*)
2. **Three zones within domain: frozen, "mushy", and unfrozen**
3. **Very useful as a numerical model benchmark because it allows for a finite freezing temperature range**
4. **Requires a 'hack' because diffusivity in mushy zone is constant**



McKenzie et al. (2007, AWR)

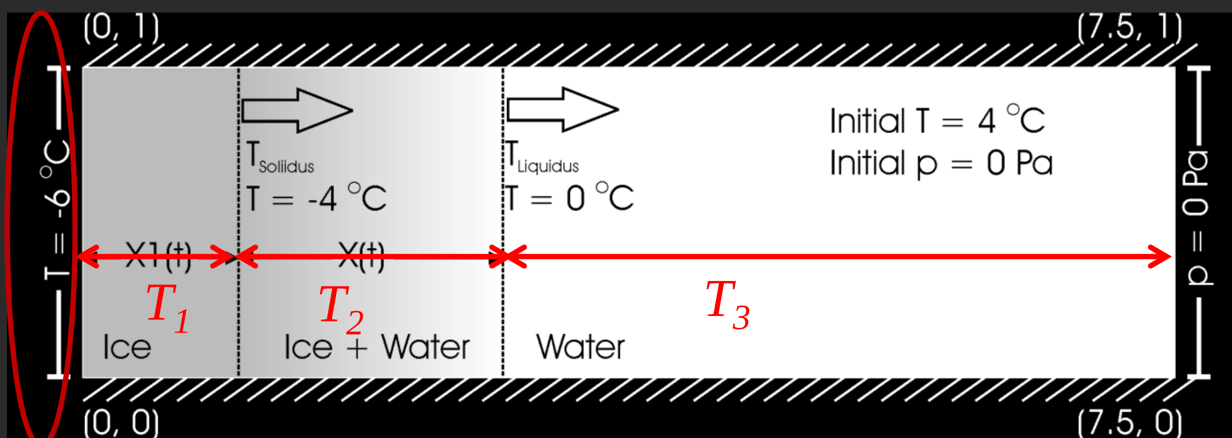
Introduction: Benchmark 1 (T1)

$$T_1 = (T_m - T_s)\frac{\operatorname{erf}(x/2\sqrt{\alpha_1 t})}{\operatorname{erf}(\psi)} + T_s$$

$$T_2 = (T_m - T_f)\frac{\operatorname{erf}(x/2\sqrt{\alpha_4 t}) - \operatorname{erf}(\gamma)}{\operatorname{erf}(\gamma) - \operatorname{erf}\left(\psi\sqrt{\alpha_1/\alpha_4}\right)} + T_f$$

$$T_3 = (T_0 - T_f)\frac{-\operatorname{erfc}(x/2\sqrt{\alpha_3 t})}{\operatorname{erfc}\left(\gamma\sqrt{\alpha_4/\alpha_3}\right)} + T_0$$

Have to solve two other simultaneous equations to obtain



McKenzie et al. (2007, AWR)

Methods: Benchmark 1 (T1)

Table 1
Parameters used in analytical solution by Lunardini [28]

Parameter	Value
T_0 (°C)	4
T_s (°C)	−6
T_f (°C)	0
T_m (°C)[a]	−4, −1
k_1 (J/s m C)	3.464352
k_2 (J/s m C)	2.941352
k_3 (J/s m C)	2.418352
C_1 (J/m^3 C)	690360
C_2 (J/m^3 C)	690360
C_3 (J/m^3 C)	690360
ξ_f (kg_{water}/kg_{solid})	0.0782
ξ_0 (kg_{water}/kg_{solid})	0.2
L_f (J/kg)	334720
γ_d (kg/m^3)	1680
γ[a]	1.395, 2.062
ψ[a]	0.0617, 0.1375

[a] Model was run for two cases, $T_m = -4$ and $T_m = -1$.

McKenzie et al. (2007, AWR)

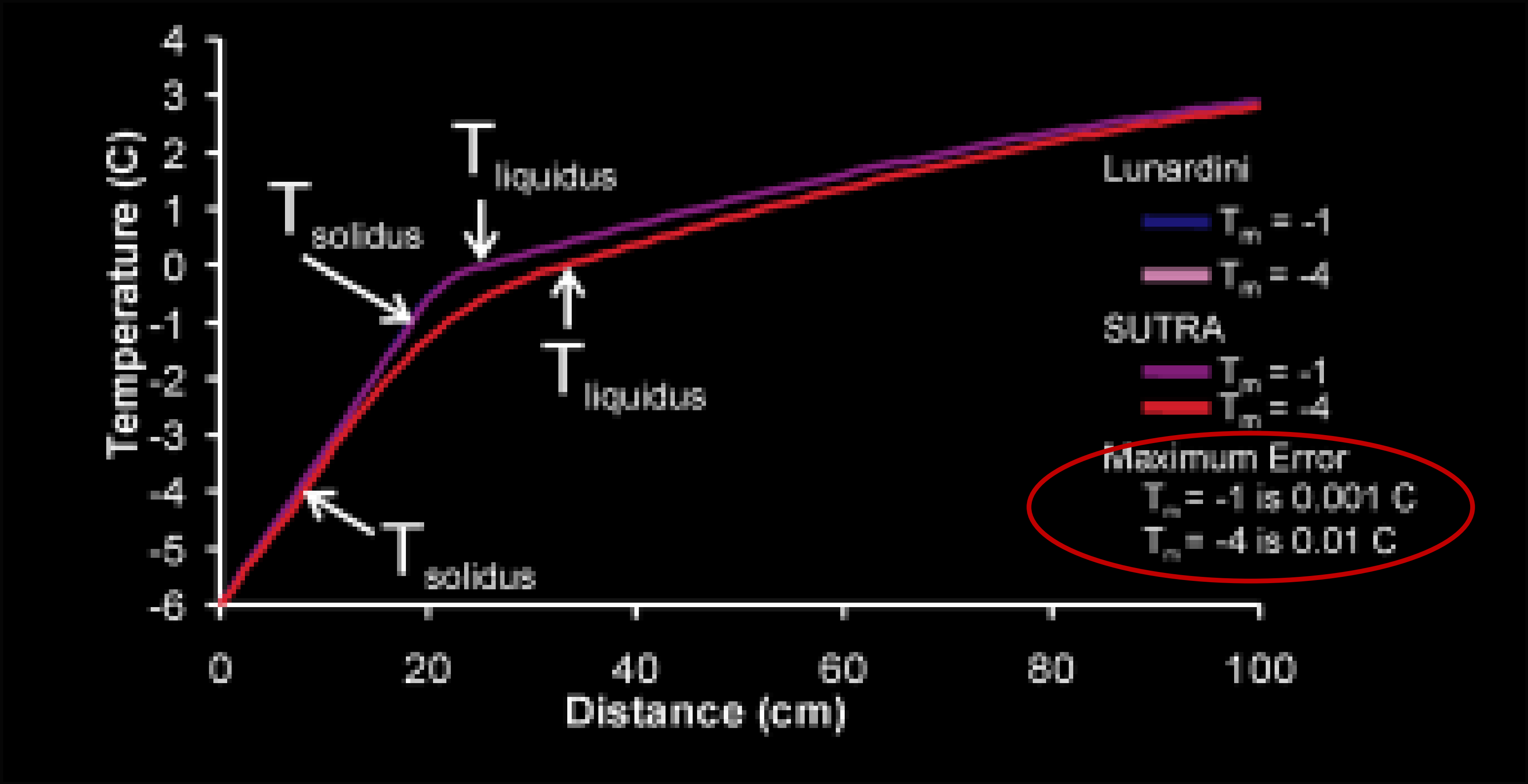
Table 2
Model parameters used in SUTRA-ICE simulations

Parameter	Lunardini
Physical properties	
Gravity (m/s^2)	0
Fluid specific heat (J/kg)	4187
Ice specific heat (J/kg)	2108
Ice thermal conductivity (J/s m C)	2.14
Density of ice (J/kg)	920
Latent heat of fusion (J/kg)	334000
Fluid thermal conductivity (J/s m C)	0.58
Fluid compressibility [kg/(m s^2)]$^{-1}$	0
Solid matrix compressibility [kg/(m s^2)]$^{-1}$	0
Solid grain specific heat (J/kg)	840
Solid grain conductivity (J/s m C)	2.9
Density of solid grains (kg/m^3)	2600
Porosity (–)	0.05
Permeability (m^2)	10^{-10}
Anisotropy ($k_{vertical}$: $k_{horizontal}$)	1:1
Longitudinal dispersivity (m)	0
Transverse dispersivity (m)	0
Time step controls	
Time step size (h)	0.25
Total simulation time (days)	1
SUTRA-ICE parameters	
Freezing function	L_{sw}
Freezing function parameters	$m = 0.25$–2.0
Residual saturation	0
Relative permeability method	L_{kr}
Relative permeability parameters	N/A
Minimum K_r	10^{-6}

McKenzie et al. (2007, AWR)

Results: Benchmark 1 (T1)

Uses temperature as the benchmarking variable

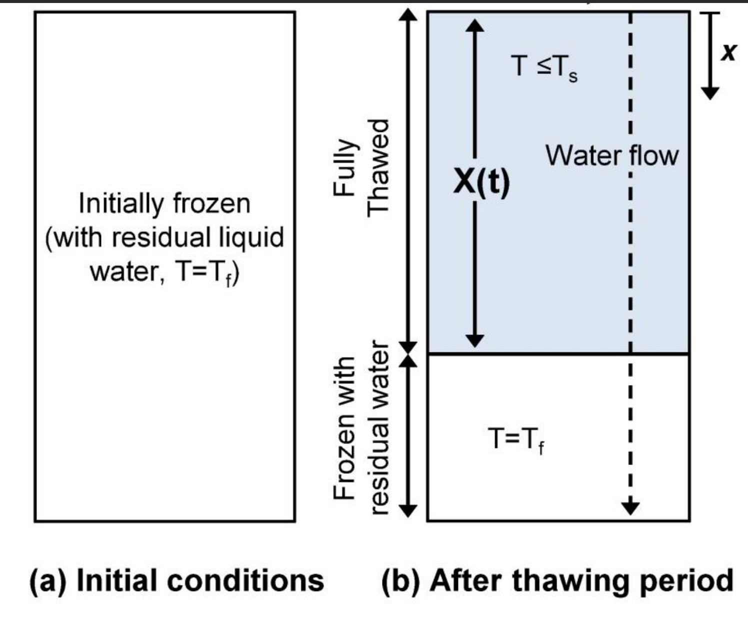


McKenzie et al. (2007, AWR)

Introduction: Benchmark 2 (TH1)

1. **Based on earlier work by Fel'dman (1972) and Lunardini (1998)**
2. **Can accommodate heat advection, and thus is an ideal benchmark for cold regions groundwater flow and energy transport codes.**

$$X + \frac{\alpha}{v_t}\left\{\exp\left(-\frac{v_t X}{\alpha}\right) - 1\right\} = v_t S_T t$$



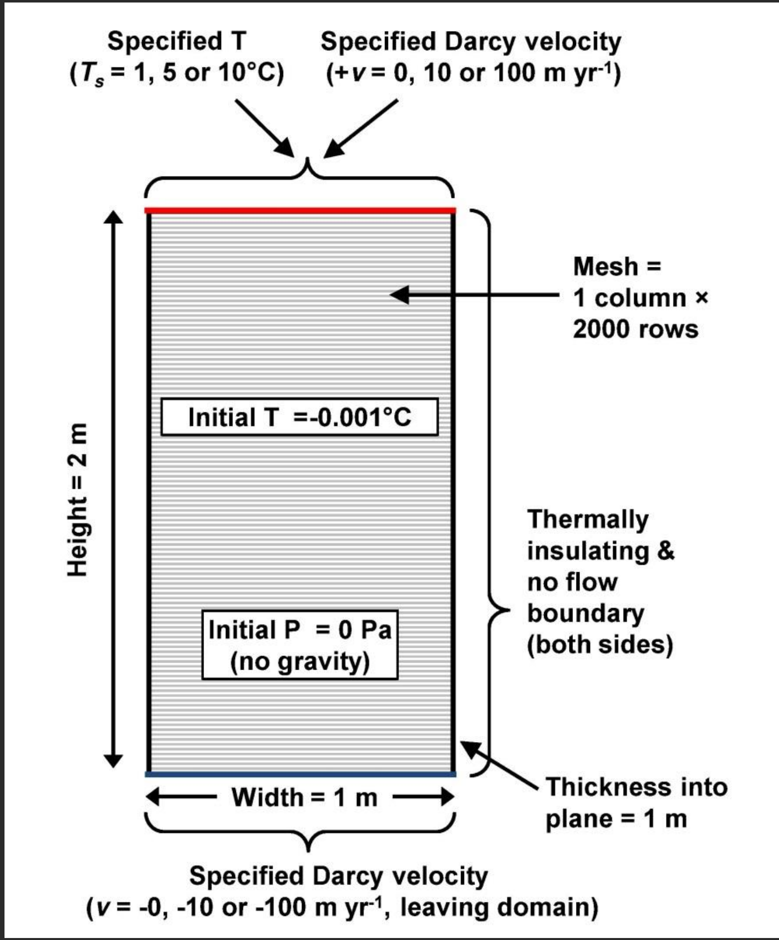
Kurylyk et al. (2014, AWR)

Uses depth to thawing front as the benchmarking variable

Methods: Benchmark 2 (TH1)

SUTRA Setup

1. **Single 2m tall column with 2000 rows**
2. **Specified flux at top (and bottom) (permeability is immaterial)**
3. **$P_i > 0$ to maintain saturated conditions**
4. **Specified surface temperature on top boundary (1°C).**
5. **Set initial temperature at or slightly below the temperature at which residual liquid saturation is first achieved (i.e. completely frozen).**



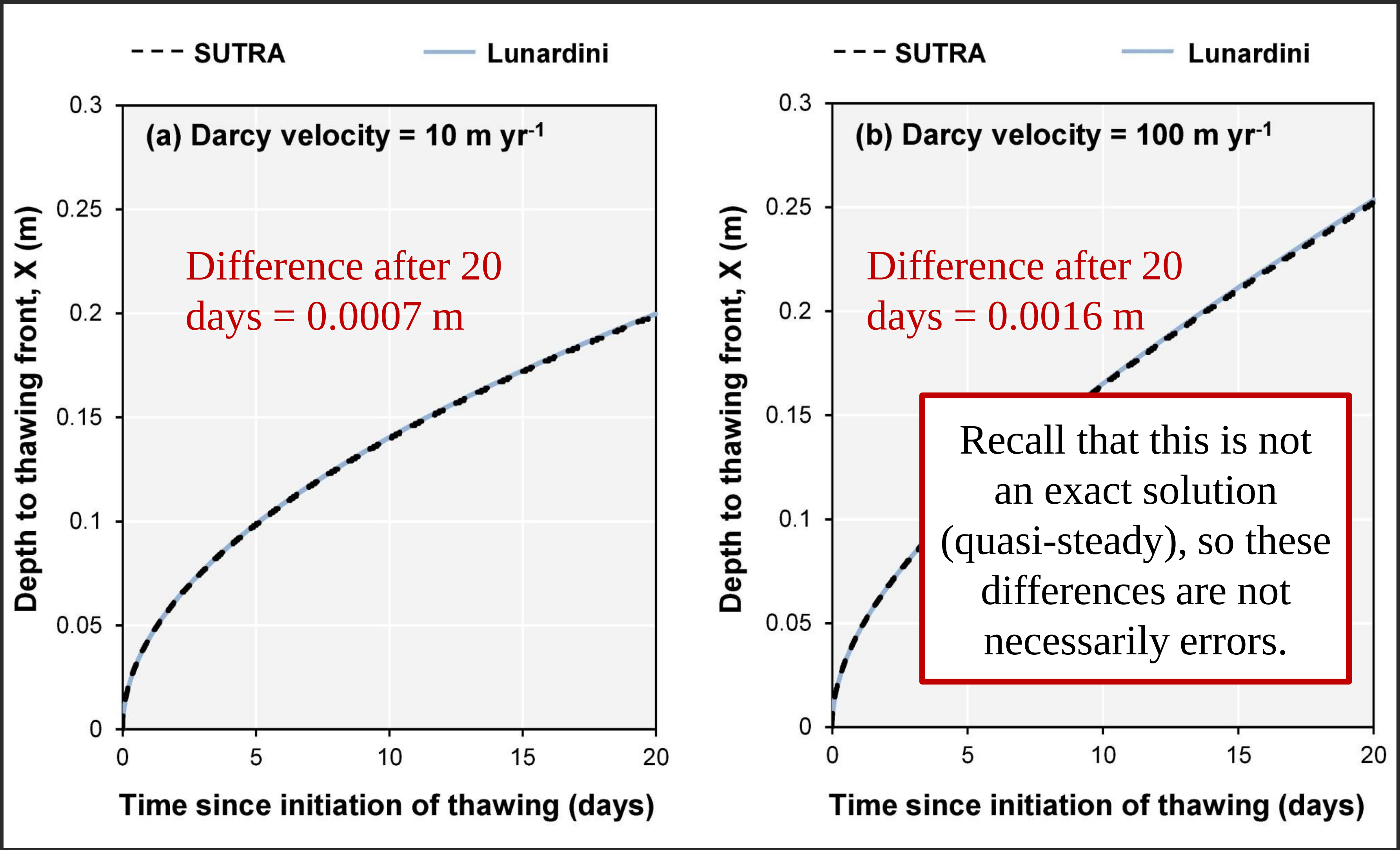
Kurylyk et al. (2014, AWR)

Methods: Benchmark 2 (TH1)

Table A1
Input parameters for SUTRA and the analytical solutions.

Parameter	Symbol	Value	Units
Hydraulic properties			
Porosity	ε	0.50 (0.25)[a]	–
Relative permeability [b]	k_{rel}	off	–
Darcy velocity (downwards)	v	0, 0.001, 10, and 100	$m\ yr^{-1}$
Gravity	g	0	$m\ s^{-2}$
Water saturation (total)	S_w	1	–
Sat. that undergoes phase change ($S_w - S_{res}$)	S_{wf}	1 (for solutions)	–
Thermal properties			
Thermal conductivity of thawed zone	λ	1.839 (2.458)	$W\ m^{-1}\ °C^{-1}$
Heat capacity of thawed zone	$c\rho$	3.201×10^6 (2.711×10^6)	$J\ m^{-3}\ °C^{-1}$
Thermal diffusivity of thawed zone	α	5.743×10^{-7} (9.067×10^{-7})	$m^2\ s^{-1}$
Thermal diffusivity of frozen zone	α_f	1.205×10^{-6} (1.297×10^{-6})	$m^2\ s^{-1}$
Thermal dispersivity	–	0[c]	m
Density of water	ρ_w	1000	$kg\ m^{-3}$
Specific heat of water	c_w	4182	$J\ kg^{-1}\ °C^{-1}$
Heat capacity of water	$c_w\rho_w$	4.182×10^6	$J\ m^{-3}\ C^{-1}$
Latent heat of fusion for water	L_f	334,000	$J\ kg^{-1}$
Other thermal settings			
Initial temperature	T_i	0[d]	°C
Freezing temperature (solutions)	T_f	0	°C
Residual freezing temperature (SUTRA)	T_{res}	−0.0005[e]	°C
Residual liquid saturation	S_{res}	0.0001	–
Slope of freezing function	b	1999.8	$°C^{-1}$
SUTRA solver settings and spatiotemporal discretization			
SUTRA element height	–	0.001	m
Number of time steps to 20 days	–	~7,000,000	–
SUTRA time step size	–	0.00001–0.0001	h

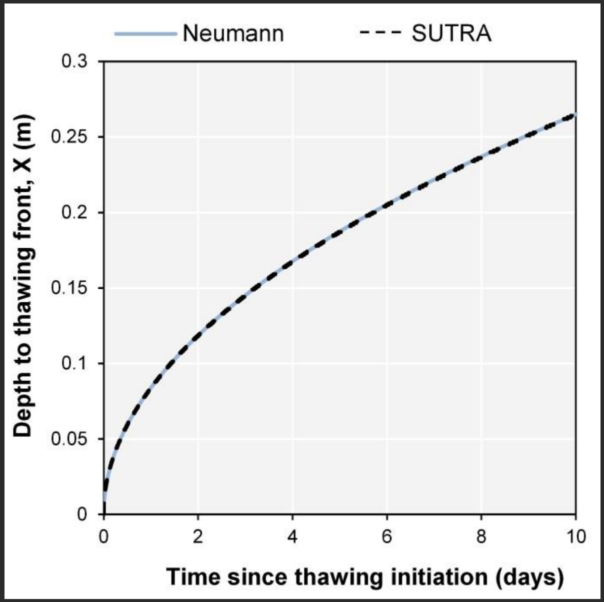
Results: Benchmark 2 (TH1)



Kurylyk et al. (2014, AWR)

Additional Comments

1. We also provide guidelines (and results) for using the classic Neumann solution as a benchmark.
2. Please send me an email if any difficulties arise.
3. Reasonable results can likely be obtained with much larger time steps.



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