Analytical solution for code benchmarking: 1D soil thaw with conduction and advection (TH1)

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UNB UNIVERSITY OF NEW BRUNSWICK

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USGS science for a changing world

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Introduction: Relevant contributions

1. G.M. Fel'dman (1972, *CRREL*)
    - Translated from Russian
    - Handwritten, illegible
2. J.F. Nixon (1975, *CGJ)*
    - Exact solution based on water sourced by thaw consolidation
    - Water flux proportional to thaw
3. V. Lunardini (1998, *7^{th} Int. Conf. PF)*
    - Exact solution (Nixon) and two approximate solutions
    - One approximate solution is inaccurate (linear temp. profile)
    - Third solution (benchmark TH1)

(Lunardini, 1998)

PERMAFROST - Seventh International Conference (Proceedings),
Yellowknife (Canada), Collection Nordicana No 55, 1998

EFFECT OF CONVECTIVE HEAT TRANSFER ON THAWING OF FROZEN SOIL

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Abstract

Most analyses of the thawing of frozen soil are based on purely conductive heat transfer, a very good assumption in most cases, but vertical and horizontal water flows occur frequently in permafrost regions. The effect of vertical water movement on the rate of thaw and the thermal regime of the soil is quantified. An exact similarity solution only occurs when the vertical water velocity is proportional to the rate of thaw. This solution indicates that seepage flows (the magnitude of the water velocity is near that of the rate of thaw) have little effect upon the thaw process. Approximate solutions are also given for the case of constant water velocity, using the heat balance integral and quasi-steady methods; they agree with the exact solution if the Stefan number is not too large. Thaw can be greatly accelerated or retarded if the water velocity (Peclet number) is large. The effect upon thawing for the case of horizontal water flow is less than that for the same magnitude of vertical flow.

Symbols

A	constant
b	exp $(v_o X/\alpha)$
c	specific heat
C	constant
f	similarity parameter
f'	df/dt
k	thermal conductivity
ℓ	latent heat
L_c	characteristic length
P_e	$L_c\, v_o/\alpha$, Peclet number
S_T	$c(T_s - T_f)/\ell$, Stefan number
t	time
T	temperature
v	velocity
x	Cartesian depth coordinate
X	thaw depth
y	$\eta - A\gamma$, dimensionless depth coordinate

Subscripts

f	freeze
s	surface
o	constant value

Introduction

The most widely used method of estimating the thaw rate in permafrost or frozen soils relies upon heat transfer by pure conduction. This is the well-known Neumann solution (Carslaw and Jaeger, 1959; Lunardini, 1991). However, the movement of water in soil systems must involve convective heat transfer along with conduction. The relative importance of the convection with regard to thaw rates is of considerable interest. Martynov (1959) and Porkhaev (1959) noted that pure conduction is the dominant mode of energy transfer in permafrost or frozen soil systems, based on qualitative reasoning about the possible heat transfer mechanisms. Johansen (1975) presented a qualitative

Remaining information from…

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Analytical solutions for benchmarking cold regions subsurface water flow and energy transport models: One-dimensional soil thaw with conduction and advection

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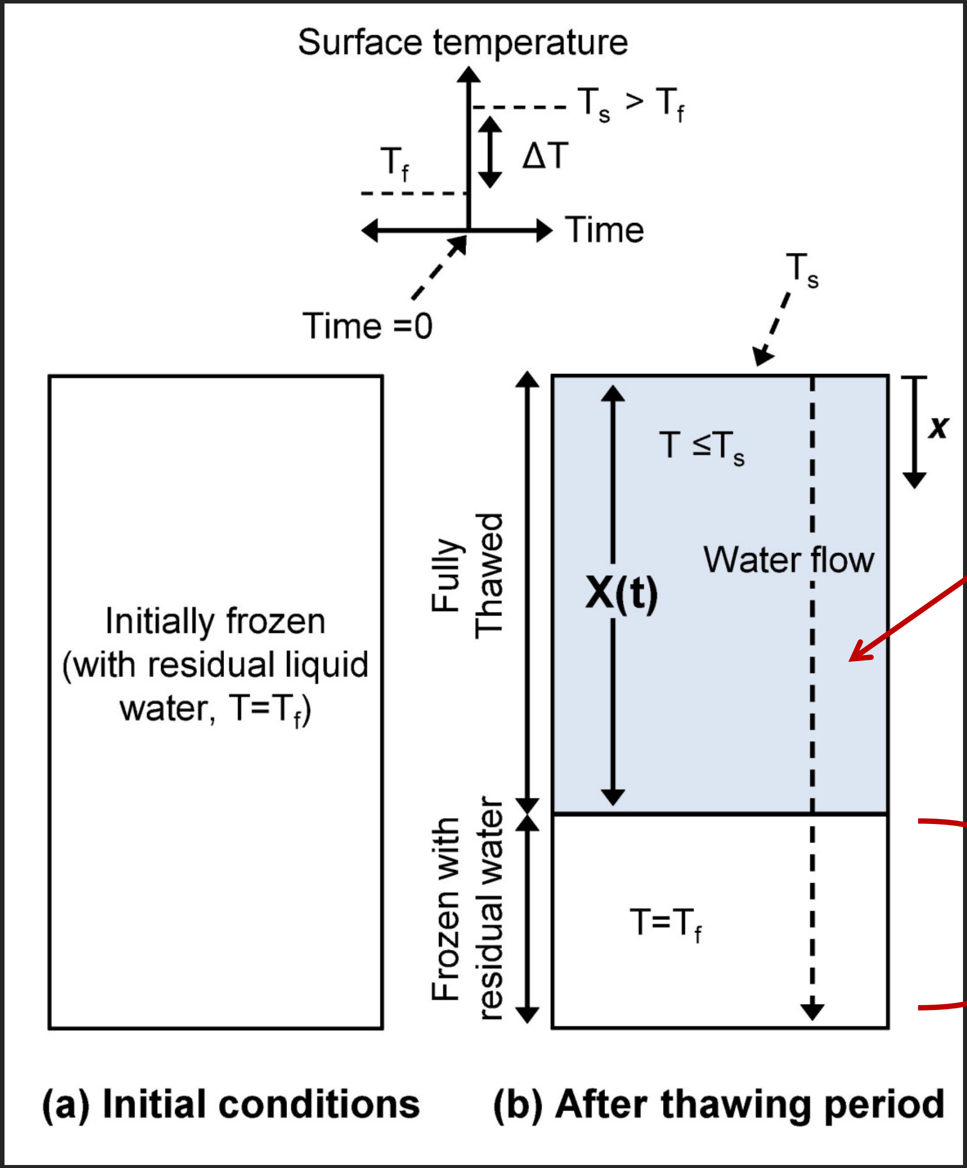
ABSTRACT

Numerous cold regions water flow and energy transport models have emerged in recent years. Dissimilarities often exist in their mathematical formulations and/or numerical solution techniques, but few analytical solutions exist for benchmarking flow and energy transport models that include pore water phase change. This paper presents a detailed derivation of the Lunardini solution, an approximate analytical solution for predicting soil thawing subject to conduction, advection, and phase change. Fifteen thawing scenarios are examined by considering differences in porosity, surface temperature, Darcy velocity, and initial temperature. The accuracy of the Lunardini solution is shown to be proportional to the Stefan number. The analytical solution results obtained for soil thawing scenarios with water flow and advection are compared to those obtained from the finite element model SUTRA. Three problems, two involving the Lunardini solution and one involving the classic Neumann solution, are recommended as standard benchmarks for future model development and testing.

TH1: Conceptual Model



Lack of fidelity to physical processes does not limit ability to serve as benchmark

Thermally uniform (no conductive flux)

Governing equations, boundary conditions, and initial conditions

1. Surface boundary condition

$$T(x=0,t)=T_s$$

2. Governing equation (thawed zone)

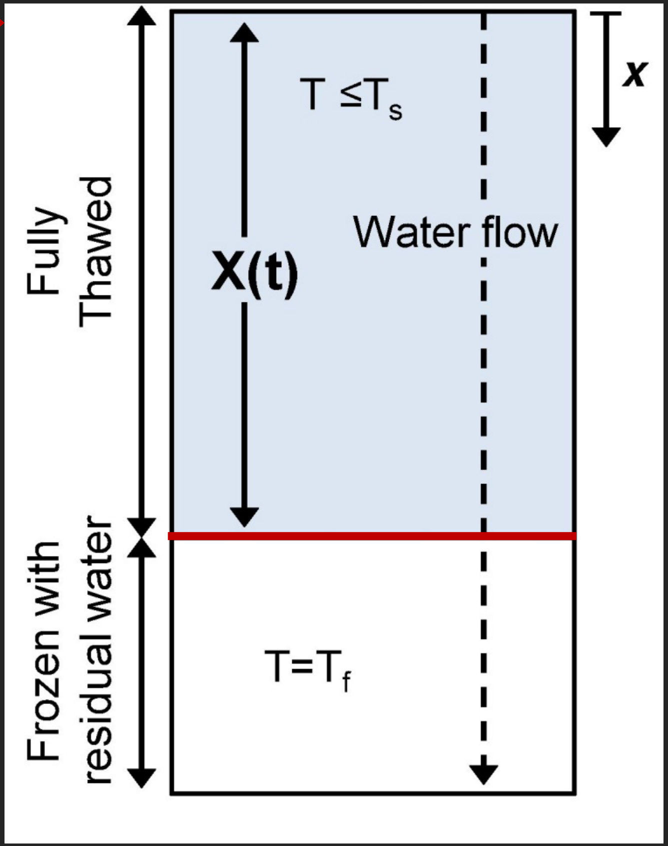
$$\alpha \frac{d^2T}{dx^2} - v_t \frac{dT}{dx} = 0$$

3. Interface energy balance

$$-\lambda \frac{\partial T(X,t)}{\partial x} + v c_w \rho_w T(X,t) = S_{wf} \rho_w \varepsilon L_f \frac{dX}{dt}$$

4. Interface boundary condition

$$T(x=X,t)=T_f=0C$$



Further thoughts on interface energy balance and resultant solution

<u>1. Kurylyk et al. (2014, AWR) energy balance:</u>

- **Lunardini (1998) ignores advection in interface energy balance**
- **Our *AWR* presentation assumes water flow only in upper layer (Eq. 19)**

$$-\lambda \frac{\partial T(X,t)}{\partial x} + v c_w \rho_w T(X,t) = S_{wf}\, \rho_w \varepsilon L_f \frac{dX}{dt}$$

(Advective flux is proportional to temperature, Saar, 2011, HJ; Kurylyk et al. 2014, ESR; Domenico and Schwartz, 1990)

<u>2. More general interface energy balance</u>

$$-\lambda \frac{\partial T(X,t)}{\partial x} + v c_w \rho_w \left[T(X,t) - T_{dat}\right] = S_{wf}\, \rho_w \varepsilon L_f \frac{dX}{dt} + v c_w \rho_w \left[T(X,t) - T_{dat}\right]$$

0°C in SUTRA

<u>3. Implicit solution (easy to derive, ~15 steps)</u>

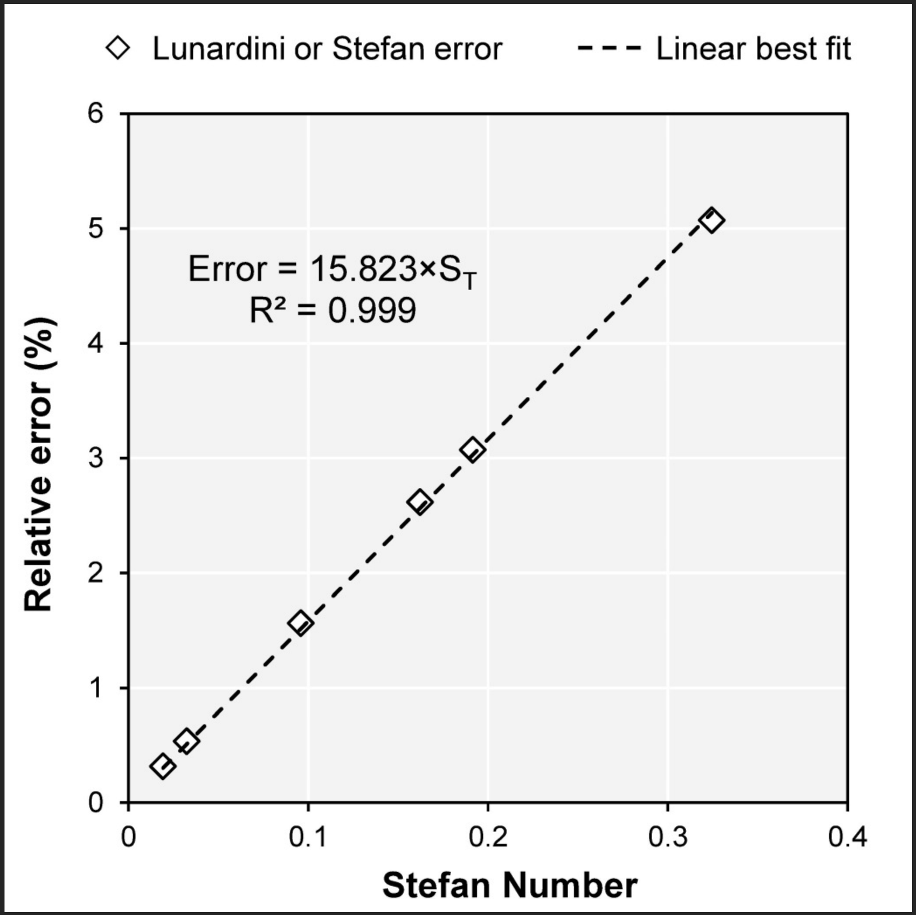
$$X + \frac{\alpha}{v_t}\left\{\exp\left(-\frac{v_t X}{\alpha}\right) - 1\right\} = v_t S_T t$$

Benchmark TH1

Accuracy of quasi-steady assumption (no flow conditions)

1. Can compare accuracy of TH1 solution by comparing it to Neumann for no-flow

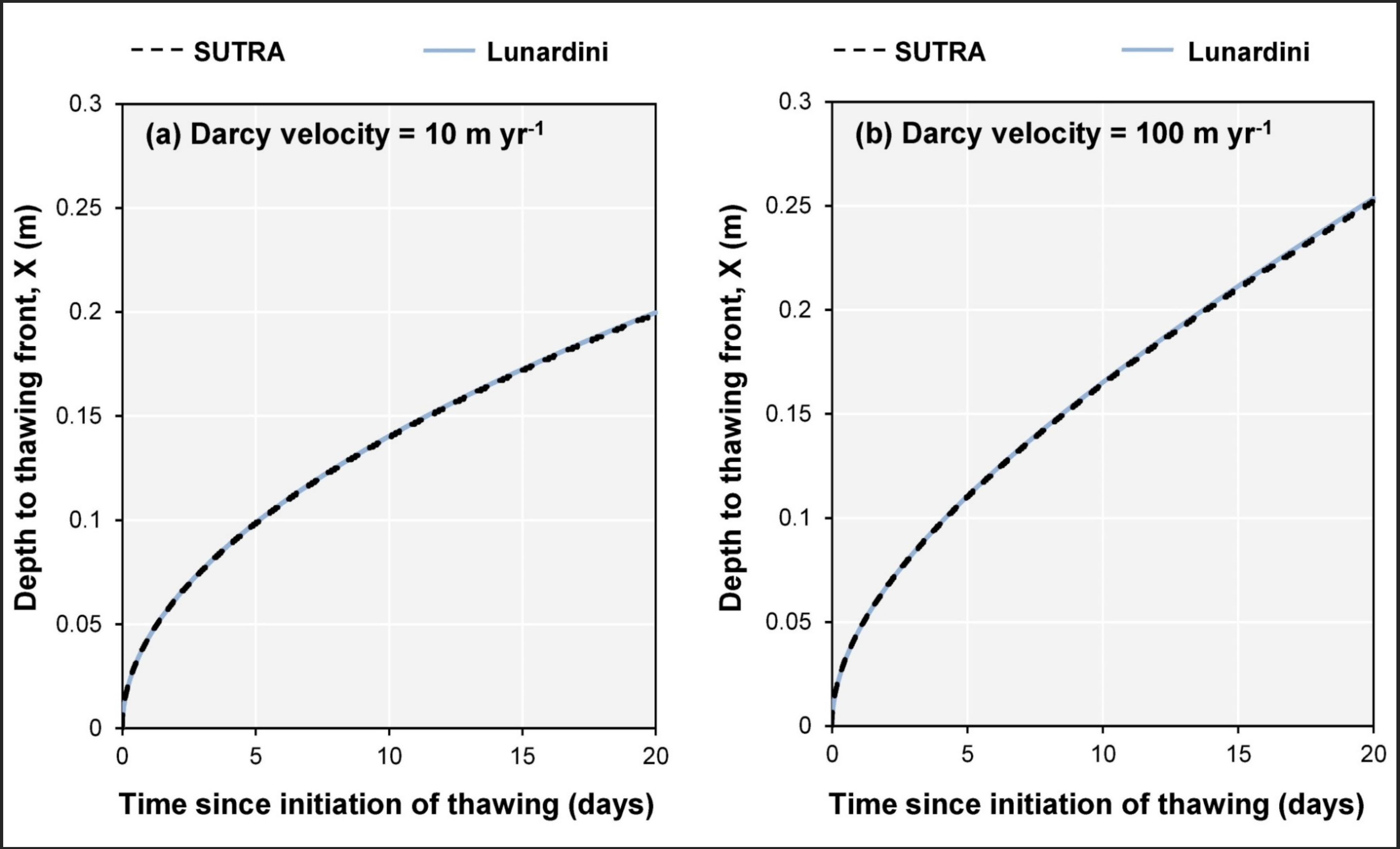
$$\text{Relative error} = \frac{X(\text{Lunardini}) - X(\text{Neumann})}{X(\text{Neumann})}$$



2. Conclusion: choose small Stefan number (surface temperature).

$$S_T = \frac{c\rho\left(T_s - T_f\right)}{S_{wf}\,\rho_w\,\varepsilon\,L_f} = \frac{\lambda\,T_s}{\alpha\,S_{wf}\,\rho_w\,\varepsilon\,L_f}$$

Results: Influence of downward water flow on thaw penetration (TH1)

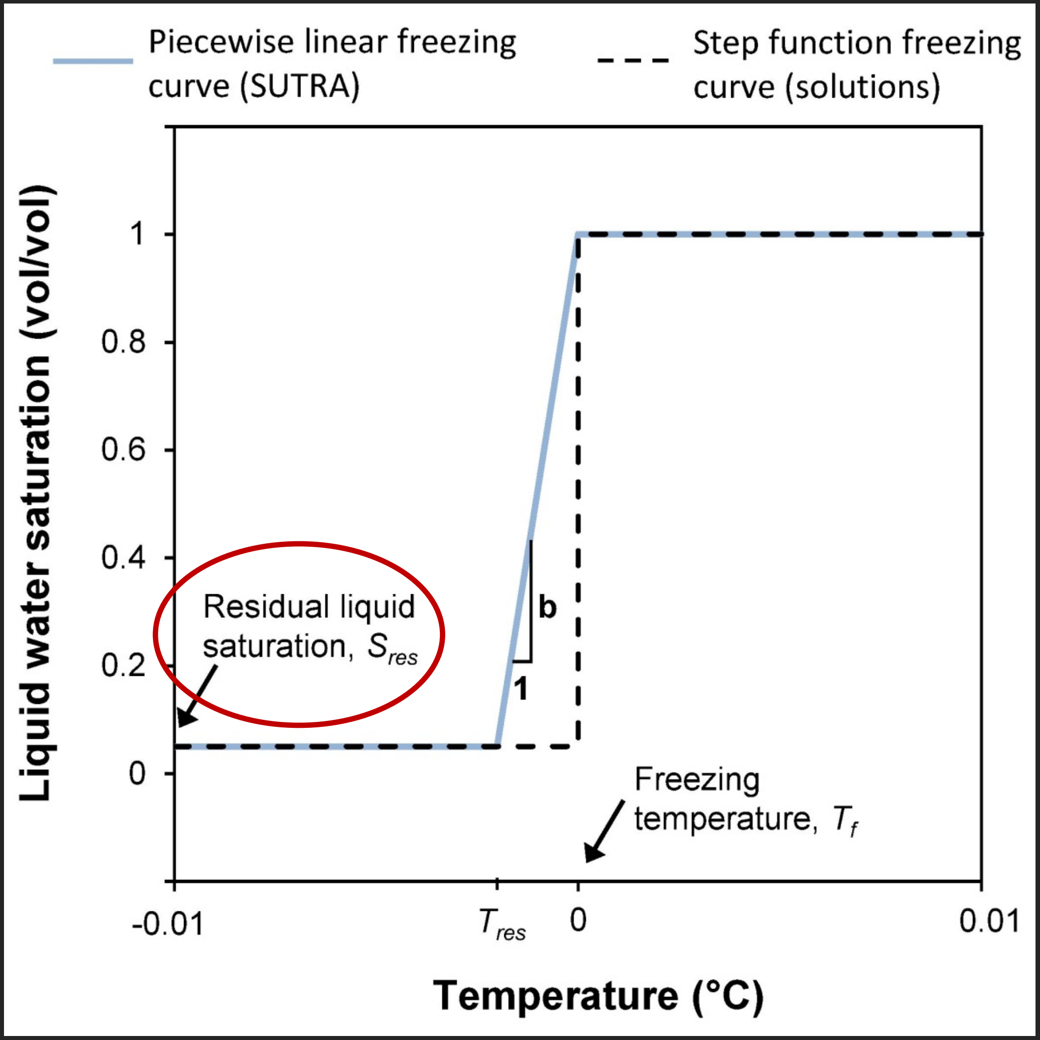


Practical considerations for reproducing TH1 benchmark

1. **Soil freezing in most solutions is a step function ($d\Theta_{ice}/dT = \infty$). Thus, use steep SFC (small time steps).**

2. **The position of 'X' can be taken as the shallowest node that is less than 0°C.**

3. **Residual liquid water must exist in frozen zone to allow for a medium for moving water (otherwise ice is advected).**



Other Research: Unsaturated Freezing

- **Unsaturated freezing is often more common than saturated freezing.**
- **There are very few comprehensive resources that address these processes.**
- **We provide a synthesis of almost a century of multidisciplinary soil freezing research.**

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Review

The mathematical representation of freezing and thawing processes in variably-saturated, non-deformable soils

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ABSTRACT

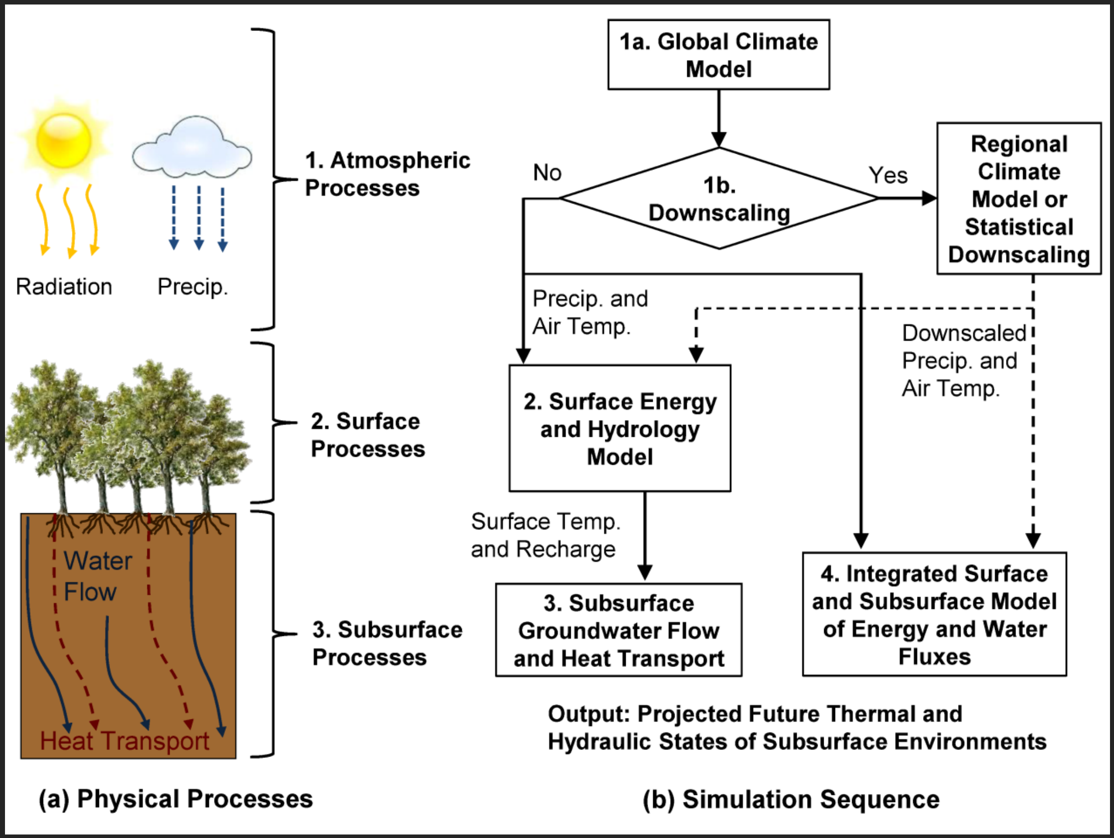
Recently, there has been a revival in the development of models simulating coupled heat and water transport in cold regions. These models represent significant advances in our ability to simulate the sensitivity of permafrost environments to future climate change. However, there are considerable differences in model formulations arising from the diverse backgrounds of researchers and practitioners in this field. The variability in existing model formulations warrants a review and synthesis of the underlying theory to demonstrate the implicit assumptions and limitations of a particular approach. This contribution examines various forms of the Clapeyron equation, the relationship between the soil moisture curve and soil freezing curve, and processes for developing soil freezing curves and hydraulic conductivity

Purpose:

1. **Explain different forms of Clapeyron equation**
2. **Explain SFC-SWC relationship (unsat)**
3. **Detail unsaturated SFCs**
4. **Discuss frozen soil permeability**

Other Research: Climate Change Impacts (2014 ESR Paper)

- Climate change impacts on cold regions hydrogeology is a major selling point for research (Kurylyk et al., 2014, Earth Sci. Rev.)
- We devote a large section to the impacts of climate change on cold regions hydrogeology.
- Our concluding section discusses future research directions.



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For more information….

Kurylyk *et al*. 2014. *Advances in Water Resources*: **70**: 172-184.

Kurylyk *et al*. 2014. *Earth-Science Reviews*: **138**: 313-334.

Kurylyk and Watanabe 2013. *Adv. Water Res.*: **138**: 313-334.

Extra Slide: 'Lunardini's' 1998 Solutions

- **Solution 1**
  - Transient heat conduction advection eq.
  - Exact solution, flux proportion to dX/dt

Velocity =C (dX/dt)

$$\alpha \frac{\partial^2 T}{\partial x^2} - v_t \frac{\partial T}{\partial x} = \frac{\partial T}{\partial t}$$

- **Solution 2**
  - Transient heat conduction advection eq.
  - Approximate analytical solution (HBIM)

Assume temperature gradient is linear

$$\alpha \int_0^X \frac{\partial^2 T}{\partial x^2} dx - v \int_0^X \frac{\partial T}{\partial x} dx = \int_0^X \frac{\partial T}{\partial t} dx$$

- **Solution 3** TH1 (INTERFROST)
  - Steady-state heat conduction advection eq.
  - For every thaw depth X, the temp. distribution is in equilibrium with surface.
  - No derivation presented

$$\alpha \frac{\partial^2 T}{\partial x^2} - v_t \frac{\partial T}{\partial x} = 0$$

Quasi-steady assumption is more consistent