Table Input parameters for SUTRA and the analytical solutions

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| --- | --- | --- | --- |
| **Parameter** | **Symbol** | **Value** | **Units** |
| ***Hydraulic properties*** |
| Porosity  | *ϵ* | 0.50  | - |
| Relative permeability a | *krel* | off | - |
| Darcy velocity (downwards) | *v* | 10, and 100 | m yr-1 |
| Gravity | *g* | 0 | m s-2 |
| Water saturation (total) | *Sw* | 1 | - |
| Sat. available for freezing (*Sw*-*Sres*) | *Swf* | 1 (for solutions) | - |
| ***Thermal properties*** |
| Thermal conductivity of thawed zone | *λ* | 1.839  |  W m-1 °C-1 |
| Heat capacity of thawed zone | *cρ* | 3.201×106  | J m-3 °C-1 |
| Thermal diffusivity of thawed zone | *α* | 5.743×10-7  | m2 s-1 |
| Thermal diffusivity of frozen zone | *αf* | 1.205×10-6  | m2 s-1 |
| Thermal dispersivity | *-* | 0 b | m |
| Density of water | *ρw* | 1000 | kg m-3 |
| Specific heat of water | *cw* | 4182 | J kg-1 °C-1 |
| Heat capacity of water | *cwρw* | 4.182×106 | J m-3 °C-1 |
| Latent heat of fusion for water | *Lf* | 334,000 | J kg-1 |
| ***Other thermal settings*** |
|  Specified temperature | *Ts* | 1 | °C |
| Initial temperature | *Ti* |  0 c | °C |
| Freezing temperature (solutions) | *Tf* | 0 | °C |
| Residual freezing temp. (SUTRA) | *Tres* | -0.0005  | °C |
| Residual liquid saturation | *Sres* | 0.0001 | - |
| Slope of freezing function  | *b* | 1999.8 | °C-1 |
| ***SUTRA solver settings and spatiotemporal discretization*** |
| SUTRA element height | - | 0.001 | m |
| Number of time steps to 20 days | - | ~ 7,000,000 | - |
| SUTRA time step size | - | 0.00001-0.0001 | hr |

a Note that because a water flux is specified at the top and bottom of the model, the actual permeability is irrelevant. For the sake of simplicity, we assumed no reduction in permeability due to pore ice formation.

b Thermal dispersivity is a parameter included in many models of coupled subsurface water and energy transport. Thermal dispersion is a thermal homogenizing process that arises due to the tortuous flow path traveled by groundwater. This phenomenon is not considered in the analytical solutions, and thus thermal dispersivity should be set to zero.

c The initial temperature for each of the analytical simulations was set to 0°C. The initial temperature could not be set at exactly 0°C in SUTRA, or the medium would be initially fully thawed. Thus the initial temperature was set at a value (-0.001°C) slightly below the residual freezing temperature Tres.